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A Simple Mathematical Model of the Innovator’s Dilemma

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Article

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Abstract

According to the innovator's dilemma set forth by Christensen (1997), incumbent companies lag behind new entrants in entering the product markets of disruptive technologies. This study presents a simple mathematical model of the innovator's dilemma (what Christensen calls the "failure framework"), incorporating factors that determine the length of the market entry time lag. The model is based on a two-firm game involving an incumbent company and a new entrant in which cannibalization within the incumbent between its conventional technology-based business and a new disruptive technology-based business influence the incumbent's decision of whether and when to enter the disruptive technology product market. The originality of the model lies in the simple formulation of temporal changes in the demand for conventional technology products and disruptive technology products through the use of a line market with vertical line at a certain time based on the diagram presented by Christensen. The model shows that a new entrant enters the market for a disruptive technology product when its initial cost of investing in the business is below a certain level; in this case the incumbent also enters the market, but lags behind the new entrant. This study also identifies economic factors that affect the range of the time lag, something that Christensen (1997) did not do. The study makes a significant contribution to research on competition in innovation and is useful for strategic decision-making by managers of both incumbents and new entrants.

Keywords: Business strategy, Christensen, Disruptive technology, Innovation, Innovator's dilemma

1 Introduction

Creating innovation is important for companies (Schumpeter, 1942). Competition for innovation has been studied by many researchers in the fields of business administration and economics (Hall, 2004; Ansari and Krop, 2012). A well-known descriptive theory that has emerged from this research is the innovator's dilemma. Based originally on case studies from the hard disk drive industry and developed by Christensen (1992; 1993; 1997), Bower and Christensen (1995), and Christensen and Bower (1996), the innovator's dilemma describes how an incumbent company which leads the market for a technological product falls behind a new entrant which uses a new disruptive technology that displaces the conventional technology and threatens the market position of the incumbent company.

Published in 1997, Clayton Christensen's The Innovator's Dilemma is viewed in academic and business circles as one of the most important books on the subject of business management of the last two decades (Greco, 2016). Despite this, no mathematical model has yet been developed of the innovator ' s dilemma, or more specifically of what Christensen called the "failure framework," which explains why firms that are dominant in an industry and well-managed often fail (Christensen, 1997, pp. xviii-xix). In this paper we formulate a simple mathematical model that describes the delay of incumbent companies in entering product markets disrupted by new technologies and derives the duration of the delay (something that Christensen (1997) did not do).

2 Literature Review

The innovator's dilemma has been studied by many business management scholars, with most studies falling into one of three categories. The first includes studies that further develop Christensen's argument by applying the innovator's dilemma concept to cases and industries beyond those that Christensen studied (e.g., Greco, 2016; Moldenhauer-Salazar and Valikangas, 2008; Mountain, 2007; Berglund and Sandström, 2017). The second category are studies that rebut Christensen's argument by providing detailed descriptions of incumbent companies successfully employing disruptive technologies to maintain their dominant market position (e.g., Appleyyard, Wang, Liddle, and Carruthers, 2008; Notarantonio and Quigley, 2013). The third category is of studies that build on the research of the second group to propose approaches to business management that allow firms to avoid the innovator's dilemma (Christensen and Raynor, 2003; Christensen, Scott, and Roth, 2004; Glazer, 2007; O' Reilly Ⅲ and Tushman, 2008, 2016; Sandström, Magnusson, and Jörnmark, 2009; Buisine, Boisadan, and Richir, 2018).

While the above-mentioned studies foster a better understanding of the innovator's dilemma and contribute to its avoidance, most are based on case studies. An exception is the statistical analysis of Igami (2017), which used data from the hard disk drive (HDD) industry to support Christensen's argument, showing that incumbent companies producing 5.25-inch HDDs fell behind new entrants making 3.5-inch HDDs by at least 57%. Igami does not offer a theoretical basis for the innovator's dilemma, however.

Numerous researchers have assessed disruptive innovation, which is the key concept underlying the innovator's dilemma. A search for this key phrase in Scopus, an electronic academic database, returned 3,910 hits in February 2021, of which 1,301 were in the field of business management and 388 were in the field of economics, indicative of the strong academic interest in studying the innovator's dilemma. As mentioned above, however, most of this research is case study-based.[1] Statistical analysis and economic modeling of disruptive innovation closely related to our work is scarce. Apart from Igami (2017), described above, Govindarajan and Kopalle (2006) developed a measure of disruptive technologies which holds potential for the development of methods that use data to verify the innovator's dilemma, and Schivardi and Schneider (2008) performed a computer simulation of a competition game between conventional technologies and disruptive technologies. These are not theoretical studies, however.

Among the few earlier theoretical studies of competition between disruptive technologies and "sustaining technologies" – Christensen's term for conventional technologies that most established companies are familiar with and that involve improving products that have an established role in the market – are Adner (2002) and Adner and Zemsky (2005). Focusing on consumer demand, Adner (2002) performed computer simulations of competition between disruptive technology-based products and conventional technology-based products. But because this study did not consider the investment behavior of incumbent companies and new entrants in disruptive technology products, it cannot be regarded as having expressed and theoretically explained the argument of Christensen (1997). Adner and Zemsky (2005) also analyzed competition between the two types of technologies using an extension of the Adner (2002) model, but this study also does not theoretically verify Christensen's argument. In contrast, the present study develops a theoretical model that expresses and verifies the argument of Christensen (1997).

As described above, earlier studies examining the innovator's dilemma and disruptive innovation have been performed by numerous researchers. Few studies, however, have taken a mathematical modeling approach to the innovator's dilemma. The absence of such a basic theory can be expected to limit future development of research in the field of innovation management. This study therefore aims to overcome that limitation. We present a model expressing the argument put forth by Christensen (1997) and develop a simple formula that facilitates understanding of the innovator's dilemma (the "failure framework"). We show how an incumbent company, which leads the market for a sustaining technology product, falls behind a new competitor in entering the market for a disruptive technology product, and consider the determinants of the duration of the new market entry delay.

3 Model

3.1 Outline of Innovator's Dilemma

Christensen summarized the innovator ' s dilemma (the " failure framework ") as follows (Christensen, 1997, pp.xvii-xxi).

[1] Studies on radical innovation in companies are closely related to disruptive innovation. Hill and Rothaermel (2003), Stoneman and Battisti (2010), and Ansari and Krop (2012) reviewed research on technological competition between incumbent companies and new entrants. Major studies of radical technological competition before Christensen (1997) include Arrow (1962), Tushman and Andeson (1986), Henderson and Clark (1900), and Henderson (1993).

1. Sustaining technologies and disruptive technologies
   Sustaining technologies increase the performance of existing products based on performance indicators that have been conventionally recognized by main customers in major markets. Disruptive technologies represent innovation that has the effect of reducing product performance based on conventional indicators, at least in the short term. Disruptive technologies offer other characteristics that are received positively by new customers. Products based on disruptive technologies are generally lower-priced, simpler, smaller, and easier to use than those based on sustaining technologies.

2. Tracking market demand and tracking technological innovation
   The speed of technological innovation at times exceeds the pace of change in market demand. This may cause companies to offer products that are "better" than what customers require, sold at a price that is higher than customers are willing to pay.

3. Disruptive technologies and rational investment
   Stable companies avoid aggressive investment in disruptive technologies, as disruptive technologies are initially accepted by the least profitable customers in the market. Companies accustomed to listening and responding to the opinions of their best customers, and thereby achieving high profitability and growth, in most cases lag behind in their investment in disruptive technologies.

In this paper we develop a simple mathematical model of the above core arguments of the innovator's dilemma.

3.2 The Logic of the Model

Christensen (1997) pointed out the tendency for incumbent companies to emphasize business based on sustaining technologies. This study develops a model that incorporates the cannibalization of an incumbent company's sustaining technology-based business by the adoption and development of products based on a disruptive technology. Hall (2004) pointed out that there are costs of adapting to a new technology (costs of investing in a disruptive technology business) as well as costs of discontinuing an existing technology that a company has developed products based on (sunk costs). The model proposed in this study therefore considers both the cost of investing in a new business based on a disruptive technology and the sunk cost of discontinuing an existing business based on conventional technology.

Christensen (1997) points out that incumbent companies often develop product prototypes based on disruptive technologies but do not move on to commercialization. As a result, dissatisfied engineers may spin off new companies to commercialize such prototypes. The cost of starting a business based on the disruptive technology is significantly lower for the incumbent company than for the new entrant; this difference is added to the sunk cost.

Arrow (1962) pointed out that returns from allocating resources to an existing technology versus allocating resources to an innovative technology are uncertain. Christensen (1997) also states that performance improvement and market size resulting from adoption of a disruptive technology are unpredictable. Therefore, we assume in formulating our model that incumbent companies make resource allocation decisions based on known profits earned in past business periods rather than by forecasting future profits earned by a new business producing products based on the disruptive technology.

3.3 Two Firms Game

Our model is built on a two-firm game. One firm, an incumbent company, produces products based on a conventional sustaining technology. The other firm, a new entrant, is a potential entrant to the market for products based on a disruptive technology. The model theoretically explains how incumbent producers of conventional sustaining technology products are slow to enter markets for disruptive technology products. We also model the time length of the market entry delay.

We first assume a case in which a new entrant enters a new market for a disruptive technology product at time $t = 0$. We next consider two scenarios: one in which the incumbent company does not enter the new market for the disruptive technology product, and one in which the incumbent company does enter that market. At time $t = t_1$, the incumbent company compares profits earned in the case of not entering the new market at all times and compares these with profits earned in the case of entering the new market during the period between times 0 and t_1. The incumbent company enters the market for the disruptive technology product when there is an opportunity loss of ($t = T$). When the incumbent company enters the new market, we assume that the market shares of the new entrant and the incumbent company are each 50%.[2]

The amount of initial investment made by the new entrant when it enters the disruptive technology product market is expressed as S_1. The total amount of sunk cost and the investment made by the incumbent company when it enters the new market is expressed as S_2. The incumbent company determines the price and performance of its sustaining technology product. The new entrant determines the price and performance of the disruptive technology product. Consumers can purchase either of the products.

The price of the sustaining technology product is expressed as p_1. The price of the disruptive technology product is expressed as p_2, where $p_1 > p_2$ according to Christensen (1997). The marginal cost of producing the sustaining technology product is expressed as c_1. The price of the disruptive technology product when the new entrant and the incumbent company enter the new market is expressed as p_3. The price and marginal cost are treated as parameters and are assumed to be $p_1 \geq c_1$ and $p_2 \geq p_3 \geq c_2$.

We consider a line market with a vertical line at time $t = t_1$, as in Figure I.1 of Christensen (1997, p. xx). We assume that the new entrant enters the market of the disruptive technology product at time $t = 0$. At this time, the line market at time $t = t_1$ moves to the right in parallel from time $t = 0$ to time t_2, when demand for the sustaining technology product of the incumbent company becomes 0.

As noted earlier, a company decides whether to enter the market of a disruptive technology product by calculating past period profit and confirming an opportunity loss. Figure 1 shows the game tree of the market entry game with the decision-making of each company.

Figure 2 was created based on Figure I.1 of Christensen (1997).

[2] The implications of the model do not change when the market share of the incumbent company is expressed as λ and that of the new entrant is expressed as $1 - \lambda$ when performing the analysis.

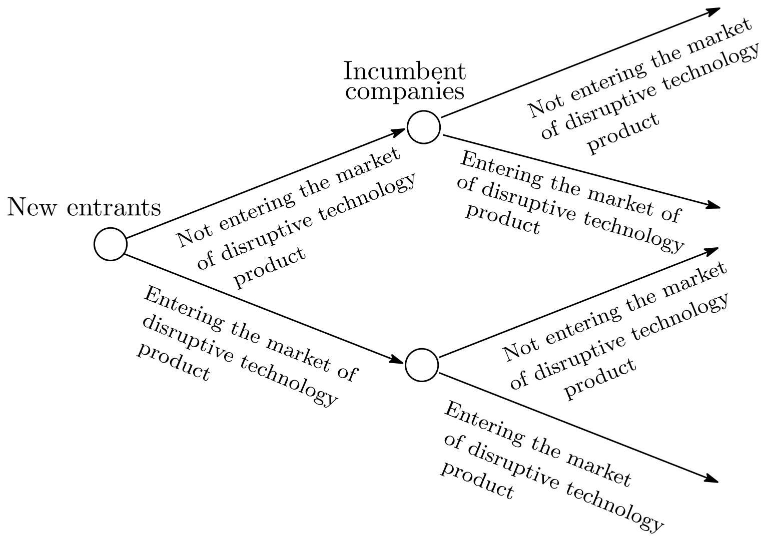

Figure 1: Game tree

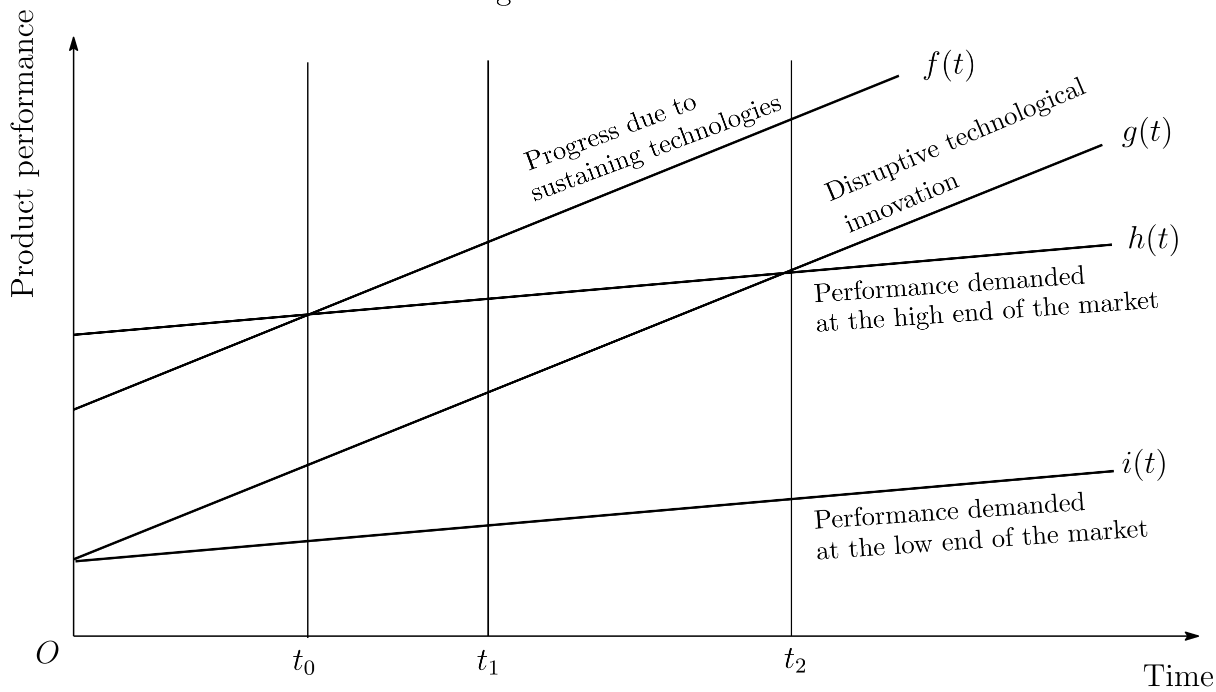

Figure 2: Effect of sustaining innovation and disruptive innovation (case of $t_0 < t_1$)

The line $f(t)$ in Figure 2 shows the performance improvement of the sustaining technology product. The line $g(t)$ depicts the performance improvement of the disruptive technology product. The technical performance of each product is assumed to improve as shown by the straight lines of $f(t)$ and $g(t)$. The price of each product is assumed to be constant. Here,

$f(t)$ and $g(t)$ are given by

$$f(t) = \alpha_1 t + \beta_1, \quad g(t) = \alpha_2 t + \beta_2,$$

where α_1 represents the speed of performance improvement of the sustaining technology product, β_1 denotes the initial performance of the sustaining technology product, α_2 stands for the speed of performance improvement of the disruptive technology product, and β_2 expresses the initial performance of the disruptive technology product.

The line $h(t)$ represents the performance improvement demanded at the high end of the market. The line $i(t)$ represents the performance improvement demanded at the low end of the market. Specifically, $h(t)$ and $i(t)$ are given by

$$h(t) = \alpha_3 t + \beta_3, \quad i(t) = \alpha_4 t + \beta_4,$$

where α_3 represents the speed of performance improvement demanded at the high end of the market, β_3 represents the initial performance demanded at the high end of the market, α_4 represents the speed of performance improvement demanded at the low end of the market, and β_4 represents the initial performance demanded at the low end of the market. As presented in Figure 2, the relations $f(t) > g(t)$, $h(t) > i(t)$, and $\beta_2 = \beta_4$ hold.

The line market with a vertical line at time $t = t_1$ does not express only the status of vertical discrimination because, according to Christensen (1997), a disruptive technology is characterized by positive assessments received from new customers. In the example of the HDD industry, the 5-inch HDD is suitable for use with a desktop PC. The 3.5-inch HDD is suited for use with a portable notebook PC. Therefore, the 5-inch HDD does not meet demand for use with a portable notebook PC. In other words, a disruptive technology product not only has a lower price and lower quality than a sustaining technology product, it also has a disruptive advantage in the form of a product feature in a different dimension. As a result, consumers purchase the disruptive technology product. The results of a questionnaire survey on disruptive technologies conducted by Govindarajain and Kopalle (2006) suggest that disruptive technologies meet the demands of consumers in market segments different from those of sustaining technologies.

The table in Christensen (1997, p. xxix) lists silver halide film as an example of a sustaining technology and digital photography as an example of a disruptive technology. The most basic quality of photographs is the clarity of the pictures. Digital photographs, however, have product characteristics other than picture clarity. These are disruptive benefits that differ completely from those of silver halide film, such as immediacy, preservability, and ease of editing. The cost of producing a digital photograph is lower than that for silver halide film (including the costs of the film, development, and printing). If the basic quality of picture clarity of digital pictures satisfies consumer demand, then a rational consumer would select digital photography, with its disruptive benefits, rather than silver halide film.

Adner (2002) and Adner and Zemsky (2005) conducted a theoretical analysis of demand structure on the assumption of the presence of an indifference curve on two dimensions: the basic quality of a product common to both sustaining and disruptive technologies, and the disruptive benefits of disruptive technology products. However, the results of their analysis do not adequately explain the consumer product purchases described in Christensen (1997). This suggests that the value of the disruptive product characteristics cannot be substituted for the high value of basic product quality in consumers' purchasing decisions. In fact, Christensen states that "when the performance of two or more competitive products has improved beyond what the market demands, customers can no longer base their choice upon which is the higher performing product" (1997, p. xxvii). Therefore, it is difficult to conduct a simple analysis on the assumption of an indifference curve with a marginal rate of substitution for these product characteristics.

This study aims to develop a model of the innovator's dilemma in resource allocation decision-making within a company. In other words, it aims to determine the time range that an incumbent company lags behind in entering the market of a disruptive technology product; is not intended to perform a detailed analysis of the nature of the disruptive technology product. We define demand for disruptive technology products and sustaining technology products and consumer purchasing behavior as follows. In a line market with a vertical line at time $t = t_1$, consumers are distributed evenly at the rate of N persons per unit of distance 1. The performance improvement line of the two technology products indicates not only the basic present performance, but the market segments that use the product attributes (e.g., the 5-inch HDD is in the market segment for use with desktop PCs and the 3.5-inch HDD is in the segment for the use with notebook PCs). The utility functions are described below.

x represents basic product quality, y represents a disruptive product's characteristics, and $\widehat{x}$ is consumer demand for basic product quality. For sustaining technology product $P1(x_1, 0)$ (price p_1, $x_1 > 0$) and disruptive technology product $P2(x_2, y_2)$ (price p_2, $x_2 > 0$, $y_2 > 0$), under the condition of $x_1 > \widehat{x}$ and $p_1 > p_2$, the utility function U_i $(i = 1, 2)$ is given by

$$U_i = \widehat{x}_i \times (1 + y_i) - p_i.$$

Demand for disruptive technology products and sustaining technology products and consumers'purchasing behavior are as follows. Consumers in interval $[i(t), g(t)]$ purchase one unit of a dis- ruptive technology product with a probability of 1. When $h(t) < f(t)$, consumers in interval $[g(t), h(t)]$ purchase one unit of a sustaining technology product with a probability of 1. When $h(t) < f(t)$, consumers in interval $[g(t), f(t)]$ purchase one unit of a sustaining technology product with a probability of 1. Therefore, the amount of sustaining technology products demanded $D_1(t)$ is given by

$$D_1(t) = \begin{cases} \Big[h(t) - g(t)\Big]N & \text{(if } h(t) < f(t)\text{)} \\ \\ \Big[f(t) - g(t)\Big]N & \text{(if } h(t) > f(t)\text{)} \end{cases}$$

The amount of disruptive technology products demanded $D_2(t)$ is given by

$$D_2(t) = [g(t) - i(t)]N.$$

The case in which neither a new entrant nor an incumbent company enters the market for the disruptive technology product is as follows. When $h(t) < f(t)$, consumers in $[i(t), h(t)]$ purchase one unit of a sustaining technology product with a probability of 1. When $h(t) > f(t)$, consumers in $[i(t), f(t)]$ purchase one unit of a sustaining technology product with a probability of 1. Therefore, demand for the sustaining technology product $D_1(t)$ is given by

$$D_1(t) = \begin{cases} \Big[h(t) - i(t)\Big]N & (\text{if } h(t) < f(t)) \\ \\ \Big[f(t) - i(t)\Big]N & (\text{if } h(t) > f(t)) \end{cases}$$

Here we consider a short delay of an incumbent company in entering the market of a disruptive technology product, and assume a zero discount rate for calculation of the sum total of discounted present value of profit streams. The the payoff matrix in the game in the case of $t_0 < t_1$ is expressed as in Figure 3.

New entrant \ Incumbent company	Not enter the market of disruptive technology product	Enter the market of disruptive technology product
Not enter the market of disruptive technology product	$0, \int_0^{t_0}[f(t) - i(t)]N(p_1 - c_1)dt$ $+ \int_{t_0}^{t_1}[h(t) - i(t)]N(p_1 - c_1)dt$	$0, \int_0^{t_0}[f(t) - g(t)]N(p_1 - c_1)dt$ $+ \int_{t_0}^{t_1}[h(t) - g(t)]N(p_1 - c_1)dt$ $+ \int_0^{t_1}[g(t) - i(t)]N(p_2 - c_2)dt$
Enter the market of disruptive technology product	$\int_0^{t_1}[g(t) - i(t)]N(p_2 - c_2)dt$ $-S_1$, $\int_0^{t_0}[f(t) - g(t)]N(p_1 - c_1)dt$ $+ \int_{t_0}^{t_1}[h(t) - g(t)]N(p_1 - c_1)dt$	$\frac{1}{2}\int_0^{t_1}[g(t) - i(t)]N(p_3 - c_2)dt$ $-S_1, \int_0^{t_0}[f(t) - g(t)]Np_1dt$ $+ \int_{t_0}^{t_1}[h(t) - g(t)]N(p_1 - c_1)dt$ $-S_2$ $+\frac{1}{2}\int_0^{t_1}[g(t) - i(t)]N(p_3 - c_2)dt$

Figure 3: Payoff matrix (case of $t_1 > t_0$)

First, we consider whether or not a new entrant enters the market of the disruptive technology product. If it does not, then the following condition is satisfied:

$$0 \geq \frac{1}{2}\int_0^{t_1}[g(t) - i(t)]N(p_3 - c_2)dt - S_1.$$

Therefore, we obtain

$$\begin{aligned} S_1 &\geq \frac{1}{2}\int_0^{t_1}[g(t) - i(t)]N(p_3 - c_2)dt \\ &= \frac{1}{2}\int_0^{t_1}[\alpha_2 t + \beta_2 - \alpha_4 - \beta_4]N(p_3 - c_2)dt \\ &= \frac{1}{2}\int_0^{t_1}(\alpha_2 - \alpha_4)tN(p_3 - c_2)dt \\ &= \frac{(\alpha_2 - \alpha_4)(p_3 - c_2)N}{4}t_1^2 \end{aligned} \quad (1)$$

When the new entrant does not enter the market of the disruptive technology product, the incumbent company receives higher profit by not entering that market. In this case, the incumbent company will not enter the disruptive product market. A new entrant enters the market of disruptive technology products when the following condition is satisfied:

$$S_1 \leq \frac{(\alpha_2 - \alpha_4)(p_3 - c_2)N}{4} t_1^2. \tag{2}$$

This implies that what determines whether or not a new entrant decides to enter the disruptive product market is the relative values of factors such as the amount of initial investment, the speed of performance improvement of the disruptive technology product, the speed of performance improvement demanded at the low end of the market, the price of the disruptive technology product, and the marginal cost of producing the disruptive technology product. The following relation holds for the timing of an incumbent company's entry into the market for a disruptive technology product.

The following relation holds for the timing of an incumbent company's entry into the market for a disruptive technology product.

$$0 \leq -S_2 + \frac{1}{2}\int_0^T [g(t) - i(t)]N(p_3 - c_2)dt.$$

Further, because $\beta_2 = \beta_4$, we obtain

$$\frac{1}{2}\int_0^T (\alpha_2 - \alpha_4)tN(p_3 - c_2)dt - S_2 \geq 0.$$

That is,

$$(\alpha_2 - \alpha_4)NT^2 - 4\frac{S_2}{(p_3 - c_2)} \geq 0.$$

Therefore,

$$T \geq 2\sqrt{\frac{S_2}{(\alpha_2 - \alpha_4)(p_3 - c_2)N}}. \tag{3}$$

It is noteworthy that the time at which demand for the incumbent company's sustaining technology product becomes zero is t_2. Obviously, T will not exceed t_2. Here, t_2 can be rewritten as follows:

$$t_2 = \frac{\beta_3 - \beta_2}{\alpha_2 - \alpha_3}.$$

Therefore, we get

$$\frac{\beta_3 - \beta_2}{\alpha_2 - \alpha_3} \geq T \geq 2\sqrt{\frac{S_2}{(\alpha_2 - \alpha_4)(p_3 - c_2)N}}.$$

This is the range of time delay when an incumbent company enters the market of a disruptive technology product. The incumbent company enters the disruptive technology product market within this range of delay and loses the competition for market share when a new entrant has first-mover advantages such as a patent advantage, a cost advantage through experience effects, or an advantage from consumers' brand preferences (Lieberman and Montgomery, 1988; 1998), if such advantages apply.

If

$$T = 2\sqrt{\frac{S_2}{(\alpha_2 - \alpha_4)(p_3 - c_2)N}}$$

holds, the delay duration depends on the speed of improvement in the performance of the disruptive technology product, the speed of performance improvement at the low end of the market, the price of the disruptive technology product both when the new entrant and when the incumbent company enter the market, the marginal cost of the disruptive technology product, and the total of the sunk cost and amount of investment when the incumbent company enters the market. Furthermore, if

$$T = \frac{\beta_3 - \beta_2}{\alpha_2 - \alpha_3}$$

holds, then the duration of delay depends on the initial performance demanded at the high end of the market, the initial performance of the disruptive technology product, the performance improvement speed of the disruptive technology product, and the performance improvement speed demanded at the high end of the market.

Recall that we have addressed the case of $t_1 > t_0$. In the case of $t_1 < t_0$, the payoff matrix is presented in Figure 4.

New entrant \ Incumbent company	Not enter the market of disruptive technology product	Enter the market of disruptive technology product
Not enter the market of disruptive technology product	$0, \int_0^{t_1}[f(t) - i(t)]N(p_1 - c_1)dt$	$0, \int_0^{t_1}[f(t) - g(t)]N(p_1 - c_1)dt$ $+ \int_0^{t_1}[g(t) - i(t)]N(p_2 - c_2)dt$
Enter the market of disruptive technology product	$\int_0^{t_1}[g(t) - i(t)]N(p_2 - c_2)dt$ $-S_1,$ $\int_0^{t_1}[f(t) - g(t)]N(p_1 - c_1)dt$	$\frac{1}{2}\int_0^{t_1}[g(t) - i(t)]N(p_3 - c_2)dt - S_1,$ $\int_{t_0}^{t_1}[f(t) - g(t)]N(p_1 - c_1)dt - S_2$ $+\frac{1}{2}\int_0^{t_1}[g(t) - i(t)]N(p_3 - c_2)dt$

Figure 4: Payoff matrix (case of $t_1 < t_0$)

As can be easily confirmed, the result derived in the case of $t_1 > t_0$ is the same as for the case of $t_1 < t_0$.

4 Discussion

This study has built a simple mathematical model that describes the core argument of Christensen (1997). As explained in the literature review, major studies preceding Christensen (1997) include Govindarajan and Kopalle's (2006) development of a measure of disruptive technologies, Adner and Zemsky's (2005) theoretical analysis of the demand structure of disruptive technologies, Adner's (2002) simulation analysis, Schivardi and Schneider's (2008) simulation of competition between old and new technologies, and Igami's (2017) verification of the innovator's dilemma using HDD industry data. None of these studies, however, take a theoretical research to verify the descriptive analytical results based on case studies presented in Christensen (1997), as this study has done.

Thus, the primary contribution of this study is presentation of a mathematical model that theoretically explains Christensen's finding that incumbent companies lag behind in entering the markets of disruptive technology products. Moreover, this study identifies economic factors that affect the time range of the market-entry delay. The originality of the model lies in the simple formulation of temporal changes in the amount of demand for sustaining technology and disruptive technology products by introducing a line market with a vertical line at a certain time based on the diagram of Christensen (1997). A secondary contribution is that this study identifies economic factors that affect the time range of the market-entry delay, something that Christensen (1997) did not do.

Key results are the following: The analysis revealed that an incumbent company certainly lags behind the new entrant when entering the market of a disruptive technology product. First, a new entrant enters the business of a disruptive technology product when the initial investment in the business of a disruptive technology product is less than a certain amount, as indicated by Equation (2). If this initial investment is greater than the certain amount (Equation (1)), then neither the incumbent company nor new entrants enter the business of the disruptive technology product. If the new entrant enters this business, then the incumbent company also enters the business of the disruptive technology product, but lags behind the new entrant. This can be regarded as the innovator's dilemma.

The maximum amount of initial investment, which is the condition for the new entrant to enter the disruptive technology product business, depends on (i) the speed of improvement in the performance of the disruptive technology product, (ii) the speed of performance improvement demanded at the low end of the market, (iii) the price of the disruptive technology product when both the new entrant and the incumbent company enter the market, (iv) the marginal cost of producing the disruptive technology product, (v) the density of consumers in the line market, and (vi) the profit calculation period.

Therefore, the new entrant can develop a disruptive technology and make a market-entry decision using these conditions as a reference.

The minimum delay of the incumbent company in entering the market of the disruptive technology product depends on (a) the speed of improvement in the performance of the disruptive technology product, (b) the speed of performance improvement demanded at the low end of the market, (c) the price of the disruptive technology product when both the new entrant and the incumbent company enter the market, (d) the marginal cost of producing the disruptive technology product, (e) the total of the sunk cost and amount of investment when the incumbent company enters the disruptive technology product market, and (f) the density of consumers in the line market. With other factors constant, if (e) is large, then the minimum delay will increase because the effect of cannibalization between businesses in the incumbent company during the delay in decision-making will increase. Furthermore, if (a) is fast, then the minimum delay will decrease because the rate of demand erosion for the sustaining technology product caused by the market appearance of the disruptive technology product is high. If (b) is large, then the minimum delay will increase because an increase in this speed causes a decrease in growth of demand for the disruptive technology product, reducing the incentive for the incumbent company to enter the market of the disruptive technology product. Moreover, if (c) is low, then the minimum delay increases because a decrease in the price of the disruptive technology product causes a relative decrease in the expected profit the incumbent company would earn by entering the market of the disruptive technology product. A decrease in (f) causes an increase in the minimum delay because a small size of the absolute market for the product causes a weakening of the incentive for the incumbent company to enter the market of the disruptive technology product.

The results presented above suggest that the factors affecting the minimum delay are related to the disruptive technology product and demand for that product, and not to the sustaining technology product and demand for it. Therefore, the entry of an incumbent company into the market of a disruptive technology product clearly lags, irrespective of technical improvement and pricing of the sustaining technology product, the level of demand, and other factors. This result is valuable information for the decision-making of incumbent company managers.

New entrants are able to obtain valuable information by comparing the conditions for them to enter the disruptive technology product market (Equation (2)) and the minimum delay of incumbent companies in entering that market (Equation (3)). The numerator of the right side of Equation (2) and the denominator of Equation (3) are the same. Therefore, even if the initial cost of investing in the business of a disruptive technology product is high for a new entrant, if the disruptive technology product market is sufficiently attractive for it to enter, then the market is equally attractive for the incumbent company. In other words, the delay of the incumbent company in entering the market of the disruptive technology product decreases in this case. Therefore, for a new entrant to have an advantage over an incumbent company in the market of a disruptive technology product, it must develop a disruptive technology product requiring a small initial investment to secure profit. That effort is crucially important, even when high demand for the product cannot be predicted before entering the market.

The maximum delay of the incumbent entering the disruptive technology market is the time until which demand for the sustaining technology product of the incumbent company drops to zero. At this point the incumbent company will no longer be able to survive without entering the new market. The maximum time delay depends on (Ⅰ) the initial performance demanded at the high end of the market, (Ⅱ) the initial performance of the disruptive technology product demanded at the low end of the market, (Ⅲ) the speed of performance improvement of the disruptive technology product, and (Ⅳ) the speed of performance improvement demanded at the high end of the market. These results also provide valuable decision-making information for managers of incumbent companies.

As presented up to this point, this study offers new and important findings by developing a theoretical model that underpins the argument of Christensen (1997). The findings obtained from this study were not clearly presented in Christensen (1997). The implications of the model described above are of academic significance as theoretical proof of the innovator's dilemma, and offer guidance for corporate managers and business operators engaging in technological competition. Christensen (1997) was limited to stating that, depending on the industry, there would be differences in the time delay of incumbent companies in entering the market of disruptive technology products; he did not explain the relationships among causal factors affecting the range of the delay, as this study does. We believe that the new findings from this study can open the door to further academic research on the innovator's dilemma and disruptive technologies.

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